

THERMO-MECHANICAL STRESS ANALYSIS OF DISSIMILAR MATERIAL JOINTS USING FEM

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Abstract: This article presents numerical investigation of isotropic dissimilar material joints. Dissimilar material joints are broadly used in various structures, including offshore, nuclear, electronic packaging, IC chip and spacecraft various fields of science and technology. In bi-material joints two different material are bonded with common interface region. High stress concentration occur at the interface of the joint under thermo-mechanical loadings due to the difference in the elastic properties and the thermal expansion coefficients of dissimilar materials. The stresses acting along the interface of dissimilar material joints are very important to determine whether the structure is reliable or not for operation. The main purpose of this research is to provide finite element solutions to predict the stress distribution at the interface of the joint based on the theory of elasticity.

Keywords: Numerical Investigation, Dissimilar material joints, Stress concentration, Stress distributions, Theory of elasticity.

1. INTRODUCTION

Recently finite element analysis have become the most convenient analyst's tool and have dealt with the real world of design engineering. Now CAD software has built-in FEA capabilities and researchers use FEA as an everyday design tool in support of the product design process.

In IC chip including SIM card bonded joints are used. Bonded joint refers to the joint in which two or more materials are joined by means of adhesive. When mechanical or thermal load acts on IC chip, large stress develops in the interface, actually near the vertex causing the bonded joint to deboned damaging the component. For this reason the electronic circuit is failed.

Strength of bi-material joint greatly depends on the orientation of physical properties and material structure. These geometrical parameters affect the performances of a bonded joint. These discontinuities may cause singularities in the stress fields or very stress concentration near the vertex of the bonding edges. This stress concentration/singularity may lead to the delamination initiation in the local area, and subsequently to the global failure of the joint structures [1].

Previously many analysis have done in the literature of the dissimilar joints. Several research studies have been conducted and reported to determine permissible stress levels, criteria of failure and material behavior at the interface of bi metallic joint. Hideo Koguchi analyzed stress singularity at three dimensional bonded joints [2]. Somnath Somadder and Md. Shahidul Islam investigated stress and displacement field of cylinder subjected to thermo-mechanical loadings by using finite element method [3]. A. Barut, I. Guven and E. Madenci determined singular stress field at multiple dissimilar material joints subjected to mechanical and thermal loading [4]. Hideo Koguchi and M. Nakajima investigated the variation of intensity of singular stress field with interlayer thickness in three-dimensional three-layered joint due to external load using boundary element method [5]. Hideo Koguchi, Yasuyuki Tsukada and Takahiko Kurahashi analyzed three dimensional singular strain field employing digital image correlation method near the corner of SI chip [6]. C. Luangarpa and Hideo Koguchi investigated 3D dissimilar material bonded joints using conservative integral one real singularity [7]. Somnath Somadder and Md. Shahidul Islam investigated stress field of a thick walled orthotropic bonded cylinder under

pressure and temperature [8]. D. Munz, A. Matthias and Y. Y. Yang determined thermal stresses in ceramic-metal joint having an inter-layer [9]. H. Pengfei, H. Ishikawa and Y. Kohno analyzed order of stress singularity at the corner of a diamond shaped rigid inclusion under bending [10].

2. ANALYTICAL SOLUTION

It is a law of nature that “matter change shapes by temperature changes.” For most materials, “Increase temperature will make its shape larger, and the reverse is true too.” Almost all metals expand with temperature increase, and contract with decrease of temperature. In general temperature, or temperature changes in a solid may induce the following effects.

1. Temperature increase will change material properties: Such as decrease the Young’s modulus and yield strength of materials.
2. Induce thermal stresses that will be added to mechanically induced stresses in solid structures.
3. Induce creep of the material, and there by make materials vulnerable for failure at high temperature.

2.1. Causes of thermal stress

There are two causes of thermal stresses in solid structures.

A uniform temperature

A uniform temperature increase in a solid rod with both ends fixed will induce compressive stress in the rod with an amount equal to:

$$\sigma = -\alpha E \Delta T, \quad (1)$$

where, α = the coefficient of thermal expansion with a unit of $1/^\circ C$, ΔT = temperature rise from a reference temperature, E =Modulus of elasticity of the material.

Solid with non-uniform temperature distributions

Stress induced by non-uniform temperature distributions of solid cause internal restraints for thermal expansion or contractions.

For two dimensional thermal stress problem, there will be two normal strains ϵ_{xT} and ϵ_{yT} along with a shear strain γ_{xyT} because of different mechanical properties in the x and y directions for anisotropic materials. The thermal strain matrix for anisotropic material is then:

$$\epsilon_T = \begin{Bmatrix} \epsilon_{xT} \\ \epsilon_{yT} \\ \gamma_{xyT} \end{Bmatrix}. \quad (2)$$

For the case of plane stress in an isotropic material with coefficient of thermal expansion α subjected to temperature rise T . The thermal strain matrix becomes:

$$\epsilon_T = \begin{Bmatrix} \alpha T \\ \alpha T \\ 0 \end{Bmatrix}. \quad (3)$$

No shear strains caused by a change in temperatures of isotropic materials, only expansion or contraction.

For this case plane strain in an isotropic material, the thermal strain matrix is:

$$\epsilon_T = (1+\nu) \begin{Bmatrix} \alpha T \\ \alpha T \\ 0 \end{Bmatrix}. \quad (4)$$

For a constant thickness and constant triangular element the thermal force matrix can be obtained such as:

The total strain energy is given as:

$$U = \int_V u_0 dV. \quad (5)$$

For a simple one dimensional rod thermal strain:

$$\epsilon_x = \frac{\sigma_x}{E} + \epsilon_T. \quad (6)$$

Let, $\frac{1}{E} = D^{-1}$ then in general matrix form equation can be written as:

$$\underline{\epsilon} = [D]^{-1} \underline{\sigma} + \underline{\epsilon}_T. \quad (7)$$

We solve for σ as

$$\underline{\sigma} = D(\underline{\epsilon} - \underline{\epsilon}_T). \quad (8)$$

The strain energy per unit volume

$$u_0 = \frac{1}{2} \underline{\sigma}(\underline{\epsilon} - \underline{\epsilon}_T), \quad (9)$$

$$\therefore u_0 = \frac{1}{2} (\underline{\epsilon} - \underline{\epsilon}_T)^T D (\underline{\epsilon} - \underline{\epsilon}_T). \quad (10)$$

The transpose is needed on the strain matrix to multiply matrices properly. Now:

$$U = \frac{1}{2} \int_V (\underline{\epsilon} - \underline{\epsilon}_T)^T D (\underline{\epsilon} - \underline{\epsilon}_T) dV. \quad (11)$$

Using $\underline{\epsilon} = B\underline{d}$ in equation (11):

$$U = \frac{1}{2} \int_V (B\underline{d} - \underline{\epsilon}_T)^T D (B\underline{d} - \underline{\epsilon}_T) dV. \quad (12)$$

Simplifying(12)

$$U = \frac{1}{2} \int_V \left(\underline{d}^T B^T D B \underline{d} - \underline{d}^T B^T D \underline{\epsilon}_T - \underline{\epsilon}_T^T D B \underline{d} + \underline{\epsilon}_T^T D \underline{\epsilon}_T \right) dV. \quad (13)$$

The first term in the equation (13) is the strain energy due to stress produced from mechanical loading – that is:

$$U_L = \frac{1}{2} \int_V d^T \underline{B}^T \underline{D} \underline{B} dV. \quad (14)$$

Terms 2 and 3 in equation (13) are identical and can be written together as:

$$U_T = \frac{1}{2} \int_V d^T \underline{B}^T \underline{D} \underline{\epsilon}_T dV. \quad (15)$$

The last and fourth term is a constant and drops out when principal of minimum potential energy is applied by setting:

$$\frac{\partial U}{\partial d} = 0. \quad (16)$$

Therefore letting $U = U_T + U_L$ and substituting equation (14) and (15) into (16) we obtain:

$$\frac{\partial U_L}{\partial d} = \int_V \underline{B}^T \underline{D} \underline{B} dV, \quad (17)$$

$$\frac{\partial U_T}{\partial d} = \int_V \underline{B}^T \underline{D} \underline{\epsilon}_T dV = \{f_T\}. \quad (18)$$

Recalling equation (3):

$$[\cdot \cdot \{ \epsilon_T \} = \alpha T]. \quad (19)$$

Thermal force matrix,

$$\{f_T\} = A \int_0^L [B]^T [D] \{ \alpha T \} dx. \quad (20)$$

For a constant thickness (t), constant-strain triangular element equation (20) can be simplified for two dimensional case as:

$$\{f_T\} = [B]^T [D] \{ \epsilon_T \} tA. \quad (21)$$

By substituting equation for [D] and [B] as follows:

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}, \quad (22)$$

$$[B] = [B_i \quad B_j \quad B_m], \quad (23)$$

Thermal force matrix becomes:

$$\{f_T\} = \frac{\alpha EtT}{2(1-\nu)} \begin{Bmatrix} \beta_i \\ \gamma_i \\ \beta_j \\ \gamma_j \\ \beta_m \\ \gamma_m \end{Bmatrix}. \quad (24)$$

2.2. Stress/Strain Relationships

Three dimensional stress/strain relationships for an isotropic body will be developed. This is done by considering the response of a body imposed stresses. The body is subjected to stresses σ_x , σ_y and σ_z independently as shown in figure.

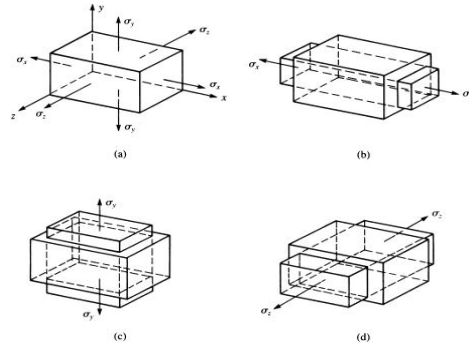


Fig. 1. Element subjected to normal stress acting in three mutually perpendicular directions

Consider, the stress in the X direction produces a positive strain:

$$\epsilon'_x = \frac{\sigma_x}{E}. \quad (25)$$

The positive stress in the y direction produces a negative strain in the x direction as a result of Poisson's effect given by:

$$\epsilon''_x = -\frac{\nu \sigma_y}{E}. \quad (26)$$

The positive stress in the z direction produces a negative strain in the x direction as a result of Poisson's effect given by:

$$\epsilon'''_x = -\frac{\nu \sigma_z}{E}. \quad (27)$$

Using superposition we obtain:

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E}. \quad (28)$$

The strains in the y and z directions can be obtained in the similar manner:

$$\begin{aligned} \epsilon_y &= -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} \\ \epsilon_z &= -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E} \end{aligned} \quad (29)$$

Solving equations (28) and (29) we obtain normal stresses:

$$\begin{aligned} \sigma_x &= \frac{E}{(1+\nu)(1-2\nu)} [\epsilon_x (1-\nu) + \nu \epsilon_y + \nu \epsilon_z] \\ \sigma_y &= \frac{E}{(1+\nu)(1-2\nu)} [\nu \epsilon_x + (1-\nu) \epsilon_y + \nu \epsilon_z] \\ \sigma_z &= \frac{E}{(1+\nu)(1-2\nu)} [\nu \epsilon_x + \nu \epsilon_y + (1-\nu) \epsilon_z] \end{aligned} \quad (30)$$

Using $\tau = G\gamma$ we obtain:

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \gamma_{yz} = \frac{\tau_{yz}}{G}, \gamma_{zx} = \frac{\tau_{zx}}{G}. \quad (31)$$

Rearranging equation (31) we have,

$$\tau_{xy} = G\gamma_{xy}, \tau_{yz} = G\gamma_{yz}, \tau_{zx} = G\gamma_{zx}. \tag{32}$$

The stresses can be expressed in matrix form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}, \tag{33}$$

where,

$$G = \frac{E}{2(1+\nu)}, \tag{34}$$

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}. \tag{35}$$

3. FINITE ELEMENT MODEL

Using ABAQUS finite element software is used for the generation of finite element model. Simulation was done by considering the effect of thermo-mechanical loads. Coupled temperature displacement step is used for stress analysis.

3.1. Physical aspects of the model

In this analysis two rectangular bars have been considered to analyze. The bars having length 10 m and width 10 m.

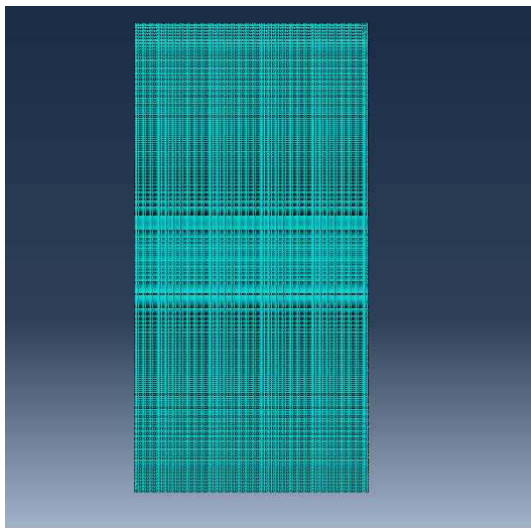


Fig. 2. Mesh of the model.

Since at the interface material property changes so bias has been used so there is more element near the interface than the other regions. As this is a 2D FEM analysis plane stress condition is considered. As this is a thermo-mechanical analysis mesh element type is used CPE8RT which is a 8-node biquadratic displacement, bilinear temperature, reduced integration.

3.2. Material properties

In this analysis, material Aluminum, and Steel is used. Here aluminum is used as upper material, steel is used as lower material.

Tab. 1. Properties of the materials used for the analysis

Material	E (Gpa)	V	K (W.m ⁻¹ .k ⁻¹)	α (°C)
Aluminium	72	0.33	234	24e-6
Steel	210	0.3	19.5	11.6e-6

3.3. Boundary Conditions

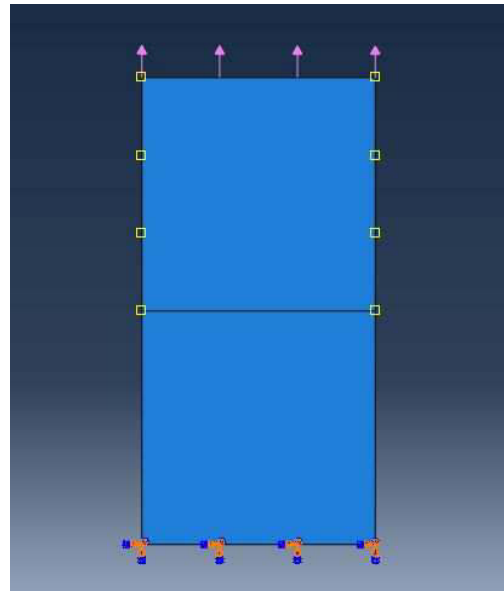


Fig. 3. Model showing boundary condition of the analysis

In this analysis the lower surface is fixed and mechanical load is applied on the upper surface. Left side of the object is at 500°C and right side of the object is at 25°C. After creating the parts and assigning material properties the parts are assembled together. By creating coupled temperature displacement step and applying proper boundary and loading conditions the problem was solved properly.

4. RESULTS AND DISCUSSIONS

All simulated results are plotted along the interface to observe stress and displacement characteristics. The graphical illustrations are presented below.

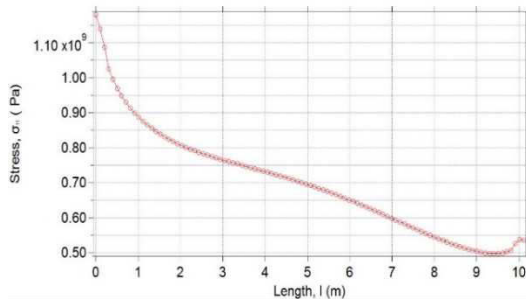


Fig. 4. Variation of σ_{11} along the length at the interface of dissimilar material joint

From the Fig. 4 variation of σ_{11} along the length at the interface of dissimilar material joint is observed. Since high thermal loadings are applied at the left side of the model high stress develops at the left region and thus satisfying the boundary condition. Very high stress occur at the interface since thermo-mechanical load is applied which may may lead to the delamination initiation in the local area and failure of the joint structures.

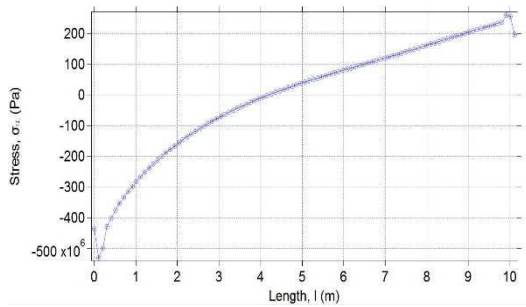


Fig. 5. Variation of σ_{12} along the length at the interface of dissimilar material joint

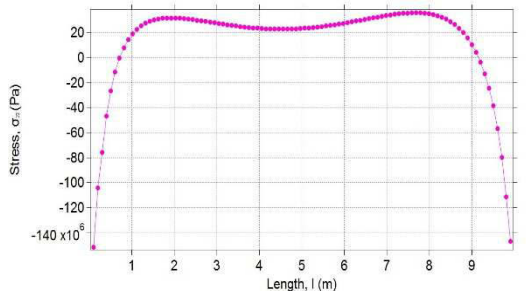


Fig. 6. Variation of σ_{22} along the length at the interface of dissimilar material joint

From the Fig. 5, Fig. 6 it is observed that distribution of normal stress σ_{22} is symmetric in nature and variation of shear stress σ_{12} is anti-symmetric in nature.

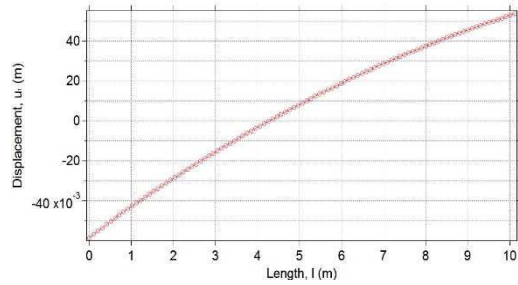


Fig. 7. Variation of u_1 along the length at the interface of dissimilar material joint

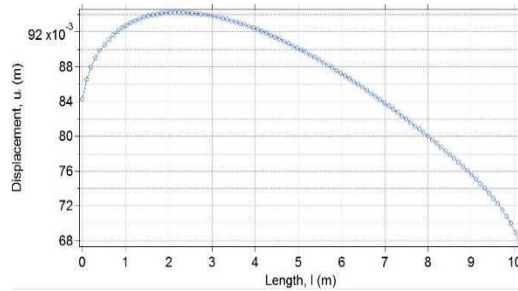


Fig. 8. Variation of u_2 along the length at the interface of dissimilar material joint

Fig. 7 and Fig. 8. indicates the variation of displacement along the length at the interface of dissimilar material joint. Variation of u_1 is quite linear in nature but distribution of u_2 varies sharply at the left side region.

From the Fig. 9 it is observed that the common interface of bi-material occurs at 10 m. The illustration indicates the continuity of stress distribution at the interface and thus satisfying the boundary conditions. It also indicates that the joint is reliable and there is no crack at the joint since there is no discontinuity at the interface of dissimilar material joint.

From the Fig. 10 it is observed that the common interface of bi-material occurs at 10 m. The illustration indicates the continuity of displacement distribution at the interface and thus satisfying the boundary conditions. It also indicates that the joint is reliable and there is no crack at the joint since there is no discontinuity at the interface of dissimilar material joint.

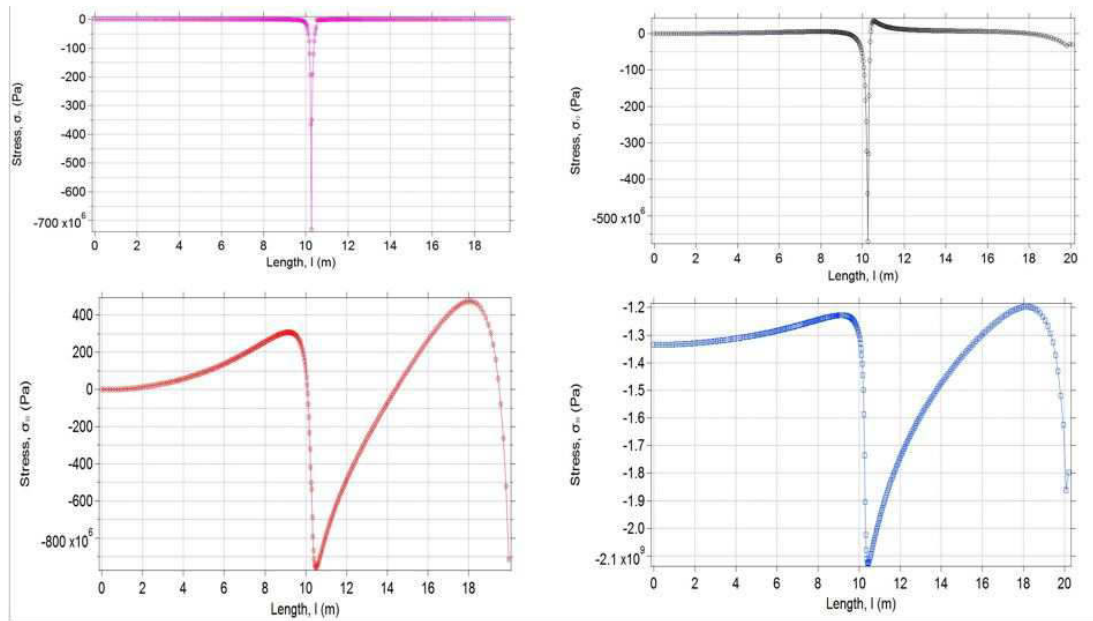


Fig. 9. Variation of stress tensor along the length showing continuity at the interface of dissimilar material joint

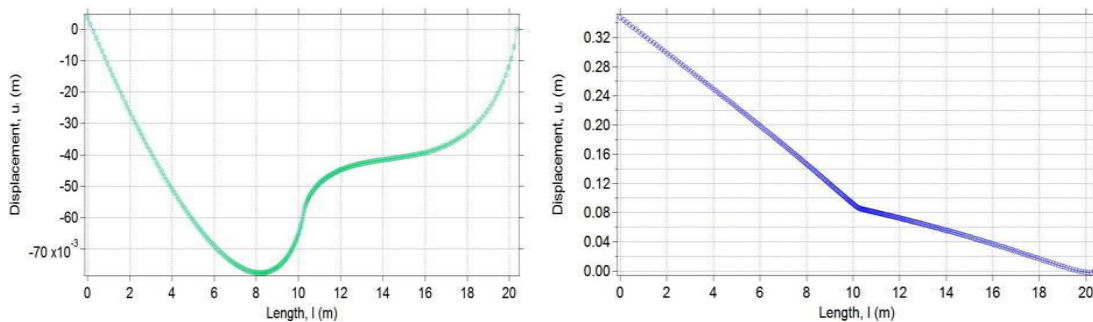


Fig. 10. Variation of displacement tensor along the length showing continuity at the interface of dissimilar material joint

5. CONCLUSIONS

The carried out research on the thermo-mechanical stress analysis of dissimilar material using FEM prove that high stress concentration occurs at the interface region than the other regions. Stress continuity is a must for a reliable dissimilar material joint. The normal stress distribution is symmetric in nature but the shear stress distribution is anti-symmetric in nature at the interface of joint. Extra care should be provided at the interface region to prevent failure as high stress concentration occurs at the interface region than the other regions. The thermal loadings have significant effect on the stress distribution at the interface of joint. The combined effects of pressure and temperature must be taken into account when designing dissimilar material joints to ensure reliability of the structure.

Nomenclature

Symbols

- $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ – Normal stress, Pa
- $\sigma_{xy}, \sigma_{yz}, \sigma_{zx}$ – Shear stress, Pa
- $\epsilon_x, \epsilon_y, \epsilon_z$ – Normal strain
- $\epsilon_{xy}, \epsilon_{yz}, \epsilon_{zx}$ – Shear strain
- E – Young modulus, MPa
- G – Shear modulus, MPa
- ν – Poisson ratio

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Biographical note



Somnath Somadder received his B. Sc. degree in Mechanical engineering (with honors) from Khulna University of Engineering & Technology in 2018 (specialization: Computational Solid Mechanics). Currently he is working as a Lecturer in Department of Mechanical Engineering, Khulna University of Engineering & Technology and doing his M. Sc. degree in same department. This manuscript is made from his research activities as a lecturer during last two years. He has published 2 scientific papers international journals, and conference proceedings. His scientific interest are focused on Analysis of stress singularity fields in 3D transversely isotropic piezoelectric and elastic bonded joints by finite element method.

